Getting $\beta-\alpha$ without penguin contributions

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Oscillation effects in $B^0 \to K_S D^0$ and related processes are considered to determine $\delta \equiv \beta - \alpha + \pi = 2\beta + \gamma$. We suggest that D^0 decays to CP eigenstates used in concert with inclusive D^0 decays provide a powerful method for determining δ cleanly. The CP asymmetry is expected to be $\lesssim 40\%$ for D^0 decays to non-CP eigenstates and $\lesssim 80\%$ for decays to CP eigenstates. This method can lead to a fairly accurate determination of δ with $O(10^8 - 10^9)$ B mesons.

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The two asymmetric B factories have made remarkable progress in determining one of the angles (β) of the unitarity triangle; the world average now stands at $\sin 2\beta^{WA} = 0.78 \pm 0.08$ [1,2]. This is in very good agreement with the expectations from the standard model (SM), $\sin 2\beta^{SM} = 0.70 \pm 0.10$ [3]. However, a considerable amount of theoretical input has to be used to deduce $\sin 2\beta^{SM}$ and progress in reducing the theory error is likely to be rather slow. Thus, methods that determine the angles without the uncertainties of hadronic matrix elements are crucial in testing the Cabibbo-Kobayashi-Maskawa (CKM) paradigm [4] to an increasing degree of accuracy in an effort to search for CP-odd phase(s) due to physics beyond the SM.

In the SM, CP violation is controlled by only one CP-odd phase. Therefore, different decays which measure the same angle of the unitarity triangle (UT) may give inconsistent results if physics beyond the SM is present. Likewise another apparent failure of unitarity of the CKM matrix, such as the failure of the UT to close, would also indicate new physics. Beyond the phase β, the determinations of α and γ, therefore, provide key SM tests.

Two extensively studied methods for determining α already exist, via $B \to \pi\pi$ [5] and $B \to \rho\pi$ [6]. In these approaches, in addition to some experimental difficulties, considerable theoretical input is essential as these modes receive large QCD-penguin as well as some electroweak penguin (EWP) contributions. While efforts at these methods should certainly continue, it is also very important that, in our drive towards precision, we develop methods that do not require theoretical assumptions and therefore have negligible theory error. The key point is that the effect of beyond the SM CP-odd phase(s) on B-physics may be quite small so any residual theory error on the determined unitarity angles may mask the effect of new physics and thwart experimental searches [7].

The processes we consider (e.g. $B^0 \to K_S D^0$) use interference between $b \to u$ and $b \to c$ tree graphs; no penguin contribution, strong or EW, is involved [8]. Time dependent CP-asymmetry measurements in $B^0(\overline{B^0}) \to K_S D^0$ will give the combination of the unitarity angles $\delta \equiv \beta - \alpha + \pi$, independent of the unitarity of the CKM matrix. If, however, one assumes unitarity of the CKM matrix then one also has, $\delta = 2\beta + \gamma$ [9]. Given that β is already well measured, this method is very effective in determining α "cleanly," i.e. without QCD complications. In addition, this method can also be used to simultaneously extract β, allowing a crucial check against the value of β determined with the $B \to J/\psi K_S^0$ approach [1,2,11]. As mentioned before, a difference in the two determinations of β may then be an indication of new physics. The basic idea behind the method has already received some attention [12–16]. We extend and complement these earlier studies in several ways so that it becomes now a powerful approach to determine α, and possibly β, without any complication from penguin contributions.

In principle a comparison of the time dependent CP asymmetry measurement in $B^0(\overline{B^0}) \to K_S D^0$ with that in $K_S \overline{D^0}$ suffices to give δ [13,15]. In practice, though, as has already been noted previously, flavor tagging of $D^0(\overline{D^0})$ appears extremely difficult [14,10]. Semi-leptonic tags suffer from very serious background from prompt B decays, $B \to l\nu X_C$; therefore, here we will not consider the possibility of semi-leptonic tags further. Hadronic tags of D^0 (say via $D^0 \to K^- \pi^+$), which are generally CP non-eigenstates (CPNES), receive appreciable corrections from doubly Cabibbo-suppressed decays of $\overline{D^0}$. As in the case of γ extraction with $B^\pm$ [10], this interference can be used to our advantage in determining δ as Kayser and London (KL) have discussed [14].

In this paper we would like to highlight at least two additional methods to enhance this style of determining δ which will be shown to have great practical importance. First of all, δ may be determined if D^0 decays to CP eigenstates (CPES) are observed, provided both $CP = +1$ and $CP = -1$ states are used. In principle, this can be accomplished if D^0 decays to one $CP = -1$ state, say K_s and π^0 and also to one $CP = +1$, say K_L and η' could be included. Of course, experimentally final states with K_L may be too hard to detect, in which case another $CP = +1$ state such as $D^0 \to K^+ K^-$ or $\pi^+ \pi^-$ can also do the job. We will also show that although one type of CPES does not provide enough information to extract δ, combining the data from one CPES with that from

D^0 decay to CPNES increases the sensitivity of getting δ over using CPNES alone.

Secondly, we will generalize these methods from single final states of D^0, $\overline{D}^0$ to inclusive sets of final states. It is important to note, though, that inclusive does not necessarily mean the sum over states with exclusive particle content. Indeed, in addition to that possibility, inclusive also means sum over different points in phase space for a single exclusive reaction, as we will elaborate below. In this way we can use the entire observable hadronic branching ratio of the D^0 greatly enhancing the statistical power. We will also briefly discuss methods whereby ancillary information constraining δ may also be obtained. The methods which we describe share with KL [14] the feature that the amplitude parameters are overdetermined and therefore a value of β, in addition to δ, may also be extracted from the same data, providing a valuable comparison to β obtained from $B\to J/\psi K_S$.

Consider now the case where $B^0(t)/\overline{B}^0(t)\to K_S D^0$, $K_S\overline{D}^0$ followed by the decay $D^0/\overline{D}^0\to F$; F denotes an inclusive set of states $F=\{f_i\}$ and in general $F\neq\overline{F}$. The set $\{f_i\}$ may range over states of different particle content (e.g. $K^- + n\pi$) or different points in phase space [17] (e.g. each f_i is a point on the $K^-\pi^+\pi^0$ Dalitz plot) or a combination of both. Thus we want to emphasize here that in this discussion we will consider inclusive any final state which is not a single quantum state even if the particle content is fixed. For instance, the decay $D^0\to K^-\pi^+\pi^0$ is an inclusive state if the result is integrated over the Dalitz plot while $D^0\to K^-\rho^+$ is an inclusive state if the polarization of the ρ is undetermined. Conversely, only a $D^0\to K^-\pi^+\pi^0$ state at a single point in the Dalitz plot is exclusive.

For each f_i (which represents single quantum states) the four relevant amplitudes are

$$\mathcal{A}_1(f_i)\equiv\mathcal{A}(\overline{B}^0\to K_S[D^0\to f_i])=A$$

$$\mathcal{A}_2(f_i)\equiv\mathcal{A}(B^0\to K_S[\overline{D}^0\to f_i])=Ar_D e^{+i\eta_D},$$

$$\mathcal{A}_3(f_i)\equiv\mathcal{A}(\overline{B}^0\to K_S[\overline{D}^0\to f_i])=Ar_D r_B e^{+i(\eta_D+\eta_B-\gamma)}$$

$$\mathcal{A}_4(f_i)\equiv\mathcal{A}(B^0\to K_S[D^0\to f_i])=Ar_B e^{+i(\eta_B+\gamma)} \tag{1}$$

where, we have adopted the Wolfenstein [18] representation of the CKM matrix, and without loss of generality we can choose the strong phase convention so that $\mathcal{A}_1=A$ is real. The quantity r_D is the ratio $|\mathcal{A}(\overline{D}^0\to f_i)/\mathcal{A}(D^0\to f_i)|$ which we will assume is known from the study of D^0 decay. The strong phase $\eta_D(f_i)=\arg[\mathcal{A}(\overline{D}^0\to f_i)/\mathcal{A}(D^0\to f_i)]$ we will assume to be not known *a priori*. Likewise the parameter r_B and the strong phase η_B given by $r_B e^{i\eta_B}=e^{-i\gamma}\mathcal{A}(B^0\to K_S D^0)/\mathcal{A}(\overline{B}^0\to K_S D^0)$ are also assumed to be not known *a priori*. Note that $\{r_D,\ \eta_D,\ A\}$ depend on the state f_i while $\{r_B,\ \eta_B\}$ are obviously independent of f_i.

The time dependent decay rates for this decay are

$$2\Gamma[B^0/\overline{B}^0(t)\to K_S F] = e^{-|\tau|}[X(F)+bY(F)\cos(x_B\tau)-bZ(F)\sin(x_B\tau)] \tag{2}$$

where $\tau=\Gamma_B t$ and $x_B=\Delta m_B/\Gamma_B$ while $b=+1$ for $B(t)$ and $b=-1$ for $\overline{B}(t)$. Defining $\mathcal{A}(f_i)=\mathcal{A}_2(f_i)+\mathcal{A}_4(f_i)$ and $\overline{\mathcal{A}}(f_i)=\mathcal{A}_1(f_i)+\mathcal{A}_3(f_i)$, the coefficients X, Y and Z in Eq. (2) are given by

$$2X(F)=\Sigma_i[|\mathcal{A}(f_i)|^2+|\overline{\mathcal{A}}(f_i)|^2];$$

$$2Y(F)=\Sigma_i[|\mathcal{A}(f_i)|^2-|\overline{\mathcal{A}}(f_i)|^2]$$

and

$$Z(F)=\Sigma_i \mathrm{Im}[e^{-2i\beta}\mathcal{A}(f_i)^*\overline{\mathcal{A}}(f_i)].$$

We can expand these quantities in terms of Eq. (1) and obtain

$$\begin{aligned}X(F)&=[(1+\hat{r}_D^2)(1+r_B^2)/2+2R_F r_B\hat{r}_D\\&\quad\times\cos(\hat{\eta}_D-\gamma)\cos\eta_B]\hat{A}^2\\Y(F)&=-[(1-\hat{r}_D^2)(1-r_B^2)/2-2R_F r_B\hat{r}_D\\&\quad\times\sin(\hat{\eta}_D-\gamma)\sin\eta_B]\hat{A}^2\\Z(F)&=[R_F r_B^2\hat{r}_D\sin(2\alpha+\hat{\eta}_D)-R_F\hat{r}_D\\&\quad\times\sin(2\beta+\hat{\eta}_D)+\hat{r}_D^2 r_B\\&\quad\times\sin(\eta_B-\delta)-r_B\sin(\eta_B+\delta)]\hat{A}^2\end{aligned} \tag{3}$$

where

$$\hat{A}^2=\sum_i A^2(f_i),$$

$$\hat{r}_D^2=\frac{\Sigma_i A^2(f_i)r_D^2(f_i)}{\hat{A}^2},$$

$$R_F e^{i\hat{\eta}_D}=\frac{\Sigma_i A^2(f_i)r_D(f_i)e^{i\eta_D(f_i)}}{\hat{A}^2\hat{r}_D}. \tag{4}$$

The corresponding quantities for $\overline{F}$ are given by $X(\overline{F})(\eta_B,\eta_D,\gamma)=X(F)(-\eta_B,-\eta_D,\gamma)$; $Y(\overline{F})(\eta_B,\eta_D,\gamma)=-Y(F)(-\eta_B,\ -\eta_D,\ \gamma)$, and $Z(\overline{F})(\eta_B,\ \eta_D,\gamma)=Z(F)(-\eta_B,\ -\eta_D,\ \gamma)$ assuming that there is no additional CP violation in D^0 decay [19].

The numerical hierarchy of the various terms in Eq. (3) depend on which kind of decay is involved. In the case of CPES decay of D^0 where $R_F=r_D=1$ while r_B, which is independent of the D^0 should be ≈ 0.36, as we will discuss below. The interference terms in the expressions for X and Y can give significant contributions while the second term in the expression for Z will dominate with the other terms making appreciable contributions. On the other hand, if D^0 decays to CPNES, r_D will be small because the corresponding $\overline{D}^0$ decay is doubly Cabibbo suppressed. In this case the interference terms in the expression for X and Y will tend to be small while the fourth term in the expression for Z will dominate. The fact that the two classes of final states are dominated by different terms in this expression suggests that combining data from CPES and CPNES will give greater leverage in the determination of δ. Below we will show in a numerical example that this indeed seems to be the case.

Let us now consider the special case where F consists of CPES with eigenvalue $\sigma=\pm 1$. In this case, the modes add coherently and so $R_F=1$, $\hat{r}_D=1$ and $\hat{\eta}_D=0$ or π for $\sigma=+1$, -1, respectively. The three observables $X(F)$, $Y(F)$, and $Z(F)$ thus depend on the four parameters $\{\hat{A}, r_B, \eta_B, \delta\}$. If we have the two data sets, for $\sigma=+1$ and for $\sigma=-1$, then there are five independent observables [note that $Y(\sigma=+1)+Y(\sigma=-1)=0$] determining the same four parameters and so the system is overdetermined and one may solve for δ.

Some examples of $CP=-1$ final states in D^0 decay [20] include $K_S\pi^0$ (BR=1%), $K_S\eta$ (0.35%), $K_S\rho^0$ (0.6%), $K_S\omega$ (1.1%), $K_S\eta'$ (0.9%), and $K_S\phi$ (0.4%) giving a total of about 4.4%. $CP=+1$ final states include K_Sf_0 (0.3%), $\pi^+\pi^-$ (.07%), and K^+K^- (.21%) for a total of 0.6%. For each of the modes with a K_S one can also construct a mode of the opposite CP by changing the K_S to a K_L. We can also change the K_S which arises from the B^0 decay, i.e. in $B^0\to K_SD^0$ (which we refer to as the fast kaon) to a K_L. Switching the fast kaon to K_L changes $\eta_B\to\eta_B+\pi$ and thus gives the same information as switching the slow kaon (i.e. the kaon arising from D^0 decay). It should be emphasized that in this instance, wherein only CPES of D^0 are considered, including the final states both with K_S and with K_L increases the number of observables and, as mentioned above, enables the system of equations to become soluble. This is in contrast, for example, with the case of $B^0\to J/\psi K_S$ versus $B^0\to J/\psi K_L$ where switching the kaon merely improves statistics but does not provide additional independent observables. Clearly detection of K_L is extremely challenging but it can help improve the determination of the angles here so experimental efforts in this direction are worthwhile; note though that K_L detection is not essential in the method [21]. Note also that much of the theory discussion goes through even when one uses another CP even final state of D^0, e.g. $\pi^+\pi^-$ or K^+K^- rather than, say, $D^0\to K_L\omega$.

We can extend this CPES method to consider inclusive final states. If F is defined in a CP invariant manner (e.g. $F=K_S+X$, BR=21%) the resultant observables will be similar to the pure eigenstate case. Here again $\hat{r}_D=1$ and $\hat{\eta}_D=0$ or π but R_F, which measures the purity of F, will not be 1. The 3 observables are thus dependent on 5 parameters $\{\hat{A}, r_B, \eta_B, \delta, R_F\}$. As before, in principle, we can obtain a solution by changing the fast K^0 to a K_L and/or changing the slow K^0, in the case where $K_S\in F$. Again, these K^0 changes will lead us to 5 observables making the system soluble. In practice, though, K_L detection in inclusive final states is likely to be too difficult so inclusive final states with K_S will need to be supplemented in some other fashion, for example, with CPNES (see below).

In [14] KL studied the special case where F consists of a single quantum state which is a CPNES (e.g. $f=K^-\pi^+$). Then $R_F=1$ but $\{\hat{A}, r_B, \eta_B, \delta, \eta_D\}$ are not known. If we take the point of view that β and all the relevant D^0 branching ratios are well determined then as discussed in [14] there are 6 observables $\{X(f), Y(f), Z(f), X(\overline{f}), Y(\overline{f}), Z(\overline{f})\}$ determining these 5 parameters and so the system is overdetermined; therefore, one can extract δ. Furthermore, as in [14] one can also take the point of view that β is a free parameter and solve for both β and δ from the same six observables. In this context, as in the CPES case, taking the fast kaon to be K_L (rather than K_S) provides 6 more independent observables dependent on the same parameters rendering the system even more overdetermined.

Since the parameters involved in the CPES case are a subset of those for a CPNES, there is a great advantage to combining the CPES and CPNES methods above. In particular, combining information from CPES and CPNES methods increases the number of observables to nine or eleven, depending on whether one or both CP eigenvalues are included. The number of parameters, of course, stays the same, i.e. five (or six if we also include β as an unknown) so that there is a considerable degree of redundancy to solve for the unknown parameters. In practice, the numerical hierarchy of the terms in Eq. (3) for CPNES final states makes such an approach very useful. For instance, we expect X and $\overline{X}$ to be similar so although we counted them as distinct observables in the above discussion, there should be a near degeneracy between them in practice. Similar comments apply to Y versus $\overline{Y}$. So combining the CPNES with the CPES becomes especially desirable as it is effective in increasing the sensitivity.

Also, considering several CPNES can enhance the degree of over determination. For each CPNES we add, we have six new observables but we introduce only one new parameter [$\eta_D(F)$], giving a net gain of 5. Indeed there are several candidate modes: $K^-\pi^+$ (branching ratio 3.8%), $K^-\rho^+$ (10.8%), $K^{*-}\pi^+$ (5.0%), $K^{*0}\pi^0$ (3.1%), $K^{*-}\rho^+$ (6.1%) and $K^{*-}a_1^+$ (7.3%) giving a total 36%. In this method one may have to separate the quasi two body modes from the broad resonances (e.g. $K^-\rho^+$) making it somewhat difficult.

Generalizing the CPNES case to inclusive states should provide the most statistically powerful data to determine δ. For instance, the inclusive $D^0\to K^-+X$ has a branching ratio of 53% [20]. In this case, we have the general case of Eq. (3) and so six observables $\{X(F), Y(F), Z(F), X(\overline{F}), Y(\overline{F}), Z(\overline{F})\}$ are determined by the six parameters $\{\hat{A}, r_B, \eta_B, r_D, \eta_D, \delta\}$ and so the system can be solved with some discrete (8-fold) ambiguities.

This may be improved in two ways. First of all, one can segregate the set F into several subsets. Thus each subset of F provides six new observables but introduces only two new parameters ($\widehat{\eta_D}$ and R_F) giving a net gain of 4 in the degree of overdetermination. This can work to a degree so long as the improvement to the χ^2 due to lifting the degeneracy compensates for the reduced statistics in the subsets of the data. For example, the inclusive mode K^-+X is made up of a

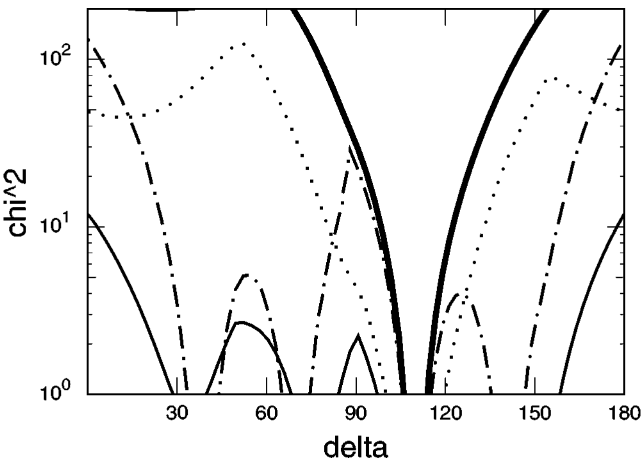

FIG. 1. The χ^2_{min} vs δ for the toy model calculation given $\hat{N}_B=10^9$. The thin solid line is the result for $D^0 \to K^- \pi^+$ alone. The dotted line is the result obtained combining $K^-\pi^+$ with CPES containing K_S. The dashed-dotted line gives the result for K^-+X alone and the thick solid line combines K^-+X with CPES containing K_S. (Note the true value of $\delta=110°$.)

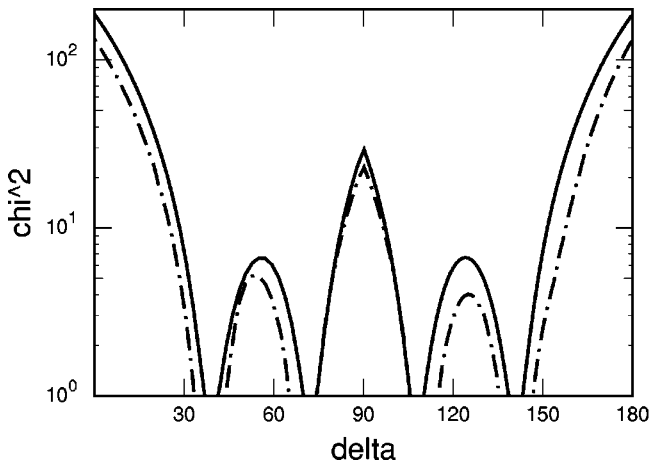

FIG. 2. Effect of r_D. For the case of K^-+X, the dashed-dotted line is as in Fig. 1 while the dashed line is the fit made with the assumption that $r_D=0$. Similarly, for $D^0 \to K^-\pi^+$, the result is the thin solid line as in Fig. 1 whereas the dotted one is the corresponding fit made with the assumption $r_D=0$.

number of different modes of distinct particle content, for instance the exclusive state $K^-\pi^+$ (4%) together with the inclusive (in the sense that these modes depend on phase space variables) states $K^-\pi^+\pi^0$ (13.1%), $K^-\pi^+\pi^+\pi^-$ (7.5%) and $K^-\pi^+\pi^+\pi^-\pi^0$ (4.0%), giving a total branching fraction of about 28%, restricting to modes containing only one π^0 in the final state. One strategy would be to consider each of these to be separate modes. Another approach would be to divide the K^-+X into separate bins according to the energy of the K^- in the D^0 frame regardless of the content of X. As an example, where such an approach is used in charged B decays see [23]. In this paper, though, we will consider getting additional data by combining the F (inclusive) method with only the CPES method.

The magnitude of the time dependent CP asymmetry for various final states can be seen in the expression for Z in Eq. (3). If $D^0 \to F$ is Cabibbo allowed then $r_D \approx \sin^2\theta_c \approx 0.05$ while $r_B \approx |V_{ub}||V_{cs}|/(|V_{cb}||V_{us}|) \approx 0.36$. The dominant term in Z thus the fourth term which will lead to CP violation of $\lesssim 36\%$. Note that this term does not become small in the limit $R_F \to 0$. In the case where F is a CP eigenstate, so $r_D=1$, the second term $\propto \sin 2\beta \approx 0.8$ becomes dominant. If R_F is small then the third and fourth terms are dominant, giving again a contribution $\propto r_B \approx 0.36$. Thus, in general, the time dependent CP asymmetry in these $B^0 \to K^0 D^0$ type of modes is expected to be rather large.

Now, in order to illustrate the relative power of the different methods, let us consider the following toy model. First, we estimate $BR(B^0 \to \overline{D}^0 K_S) \approx \sin^2\theta_c (1/N_c^2) BR(B^0 \to D^-\pi^+)/2 \approx 10^{-5}$. For this example, we will arbitrarily take $\eta_B=50°$ and $\eta_D=70°$ with $\gamma=60°$ and $\beta=25°$, consistent with the B factory values [3] and so $\delta=110°$.

In Fig. 1 we plot the χ^2 which would be obtained assuming that $\hat{N}_B=(\textit{number of B mesons})(\textit{acceptance})=10^9$ for various combinations of the data [24]. The thin solid line, which assumes a sample of about 65 such events, gives the minimum value of χ^2 as a function of δ obtained for the data from the single final state $K^-\pi^+$. Clearly discrete ambiguities in the solution tend to conspire to keep the value of χ^2 relatively low. The dotted line shows the case when $K^-\pi^+$ data is combined with CPES containing K_S. The sample of CPES events is thus contains 90 events. The dashed-dotted line gives the result for the inclusive K^-+X alone where we have taken for the purposes of illustration $\hat{\eta}_D=70°$, $\hat{r}_D$ is the same as for the $K^-\pi^+$ and $R_F=0.1$ and the thick solid line combines this with the $CP=-1$ eigenstates. In all cases, we have assumed that the overall tagging efficiency for the B^0 flavor is 25%. Thus the graph assumes 750 K^-+X events.

Experimentally, the most effective way to observe D^0 decays of the form K^-+X is to add together the separate decay rates for each particle content of the form $K^-+n\pi$. The advantage of analyzing multibody modes as inclusive modes is that the analysis is then independent of any assumptions concerning the structure of the amplitude.

In Fig. 2 we plot the K^-+X as in Fig. 1 (dashed-dotted line in both cases) and also show the fit we would obtain if we were to interpret the experimental data assuming that $r_D=0$ (dashed line); we can see that there is not a large difference in the behavior of the χ^2 under these two interpretations, in this instance. However, for the case of $D^0 \to K^-\pi^+$ (thin solid line versus dotted line), even though r_D is small neglecting it makes a significant difference; with $r_D=0$ the χ^2 always seems to stay quite large making it difficult to find a solution.

Table I shows the one sigma error on δ for various inputs. Clearly, the best results are obtained when the observables overdetermine the parameters and a large fraction of the BR is included in the sample. Thus when K^-+X is used together with K_S CPES, error on δ is $\pm 2.5°$ with $\hat{N}_B=10^9$; with $\hat{N}_B=10^8$ this error increases to $\pm 11.4°$, so is still quite useful.

Since overdetermining the system of equations is key to

TABLE I. Attainable one sigma accuracy with various data sets given $\hat{N}_B=10^9$.

Case	Accuracy
CP-final states with K_S and $CP+$ states, i.e. $\pi^+\pi^-$, $K_S f_0$ and K^+K^-	$\pm 18.2°$
The CPNES $K^-\pi^+$ together with CPES, both with K_S only	$\pm 9.0°$
CPNES $K^-\pi^+$ with K_S and with K_L	$\pm 5°$
K^-+X together with K_S CPES	$\pm 2.5°$

improving accuracy on δ it may be useful to introduce additional constraints. First of all, one can replace the D^0 and the K_S with higher resonances which will tend to increase the total statistics. In addition, as suggested in [14] if the D^0 is replaced with a D^{0**} then we can tag the flavor of the D^{**} through the decay $D^{0**}\to D^+\pi^-$. The analysis of decays with this tag thus reduces to that of [13,15]. There is the practical problem implementing this method that one must separate the D_1 and D_2 states which are 40 MeV apart. Secondly, as suggested in [10], using the methods of [22,23] one can directly determine R_F and $\hat{\eta}_D$ from studies at a $\psi(3770)$ charm factory.

Finally, the technique discussed here for replacing a single state with an inclusive one in the interference of two amplitudes has an immediate application to getting γ. In [23] we consider this for the method of [10] for extracting γ from $B^-\to K^-D^0$ with various D^0 final states. In particular, this gives a model independent way of analyzing three-body B and D final states.

In conclusion, we show that the ability to determine δ through $B^0\to D^0K_S$ can be greatly enhanced by considering D^0 decays to CP eigenstates and by using inclusive sets of D^0 decays. In particular, using inclusive D^0 decays such as K^-+X together with CP eigenstates, our illustrative calculation suggests that as the number of available B mesons increases from 10^8 to 10^9 a determination of δ with a one sigma error of $\lesssim\pm 11.4°$ becomes feasible even with a modest acceptance of $O(10\%)$. This error can, of course, be reduced to the level of a few percent as the acceptance is improved. The method described to make use of inclusive states is likely to have a wider application to the extraction of CP violating phases.

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